I. INTRODUCTION

One of the first topics that physics students learn is the acceleration of bodies falling toward the center of the earth. The magnitude $g$ of the acceleration due to gravity is usually determined in undergraduate laboratories by measuring the period of a simple pendulum. Some students wonder why the acceleration due to gravity is not determined by measuring the time taken by a freely falling body to fall through a given height. However, the body has to drop an appreciable distance to provide a measurable time of flight that is significantly larger than the measurement error.

Over the last three decades various experiments have been designed to measure the time of flight in laboratory conditions.\textsuperscript{1–4,6–10} Such a measurement is now possible due to the advent of inexpensive electronic circuitry capable of measuring time accurately in milliseconds. In most of these experiments the body is released by either a lever or electromagnet. The position of the release mechanism is taken as the timing circuit when the body crosses it. The acceleration due to gravity is calculated using the relation

$$s = v_0 t + \frac{1}{2} gt^2.$$

This setup introduces an error in the time $t$ due to the uncertainty in the time of release. In Ref. 4 an error was introduced due to the body having a nonzero velocity by the time it arrived at the first sensor that initiated the release.

In all the proposed experiments there was an error either in the measurement of the time of fall or the initial velocity. These experiments also rely on the validity of Eq. (1). To overcome this problem a method was devised to allow a metallic ball to drop between two parallel wires in which high tension current flows in pulses. Sparks generated by the traveling ball are recorded on a strip of wax paper. In another method two test masses are required.\textsuperscript{1}

We have designed a simple setup to measure the acceleration due to gravity such that very little skill is required and the validity of Eq. (1) is not assumed. In the following we discuss our setup, which uses LEDs, photodiodes, op amps, and a microprocessor.

II. APPARATUS

A hollow tube (which we shall refer to as the drop tower) of length 1 m houses eight LEDs (light emitting diodes) and eight LDRs (light dependent resistances). They are arranged along the length of the drop tower facing each other (see Fig. 1). The distance between two neighboring LEDs (and the LDRs) is 10 cm. The first LED-LDR pair is 10 cm from the top edge of the drop tower to make sure that stray light from the room does not cause false triggering of the circuitry.

As the name suggests, the resistance of LDRs depends on the amount of light falling on them and hence they are popularly used as optical sensors. When light is incident on the LDR, its resistance is low. When a falling object cuts the light’s path, no light falls on the LDR and its resistance increases. This variation in the resistance can be converted into voltage pulses. The required circuitry is shown in Fig. 2. The LDR resistance is converted to a corresponding voltage using a voltage divider circuit. The voltage across the resistance, $R_L$, is $V_H$ when light falls on the LDR because its resistance decreases. Conversely, if the LDR is in a low light region, its resistance is high and the voltage across $R_L$ is $V_L < V_H$. This voltage is sent to the op amp comparator circuit. The variable voltage is set at a value between $V_L$ and $V_H$ and ensures that the output of the LM741 op amp is +12 V when light falls on the LDR and −12 V when a shadow falls on it. This voltage is made TTL/microprocessor compatible (varies between 0 and +5 V) using a 4.7 V zener diode.\textsuperscript{5}

The final block of our setup is a IC8085 microprocessor training kit. To follow in detail requires an understanding of the IC8085.\textsuperscript{11} A standard 8085 microprocessor training kit comes with a programmable input/output device (IC8155) that has as many as 22 input pins divided into port A with eight pins, port B with eight pins, and port C with six pins. The output of each wave-shaping circuit can be fed to one of these pins. We used only eight pins of port because we wanted to minimize the amount of hardware (wave-shaping circuits) and eight data points are sufficient to fit a curve reliably. The circuit can easily be extended by removing the IC8155 to interface with a computer because the computer’s parallel port can only accept eight bit data.

When no object cuts the light path, all the pins of port A are high and the microprocessor is informed by sending the word FFH. (The subscript H is the hex representation of the binary number 1111 1111.) This word changes when an object cuts the light’s path. For example, on cutting the first
LED light’s path the word sent to the microprocessor is 1111 1110 (FEH), and when the object cuts the second LED-light’s path the microprocessor receives the word 1111 1101 (FDH), and so forth as the object falls. The microprocessor is programmed to run a counter from the negative edge of one input signal to that of the second (see Fig. 3). On receipt of the second input signal’s negative edge, the microprocessor stores the counter’s count in its memory and restarts the counter. This procedure contributes an error the measured time is less than the actual time in the measurement of the time of flight between the two adjacent LED-LDR pairs. The counts are converted to time in seconds using

\[ t_1 = (30n + 34) \times 0.326 \mu s, \]  
\[ t_2 \text{ to } t_8 = (30n + 41) \times 0.326 \mu s. \]

Equation (2) depends on the number of instructions the program executes in the loop for detecting the signal’s negative edge. Because it takes \( 7 \times 0.326 = 2.282 \mu s \) to write in the microprocessor’s memory location,\(^{11}\) the expression for \( t_2 \) to \( t_8 \) is different than that for \( t_1 \) because the program has extra

\[ \text{instructions for storing in memory the number of loops between two negative edges. Note that both the wave-shaping circuit and the software are designed to minimize any delay in the response by the cadmium sulphide LDR. The assembly language instruction set used for the project can be downloaded.}^{12} \]

### III. RESULTS AND DISCUSSIONS

The drop tower was kept vertical and an attempt was made to ensure that the objects fall in a straight line. A plumb line was used to determine the alignment. We used three balls whose features are given in Table I. The diameters of the balls are much less than the tower’s cross-section of 10 cm to ensure that the walls of the tower played no role in the motion of the objects.

The balls were dropped from the top edge of the drop tower. We made sure that the ball would cut all eight light beams. As soon as the ball crosses the first LED, the microprocessor starts its clock. This point corresponds to the coordinates \( (0, 0) \). The microprocessor stores the counter’s count as the ball passes the second LED the count is used to calculate the time. This count gives the coordinates \( t_{21}, 0.1 \) where the subscript 21 indicates the time taken for the fall between the first and second LED. Because the LEDs are 10 cm apart, \( s = 0.1 \) cm. In this manner, eight data points are generated and stored in the microprocessor. The experiment is repeated as many as 200 times for each ball.

Figure 4 shows the results obtained with ball 1 and ball 2. The data lie on a parabola. However, we have to remember that Eq. (1) represents an object falling freely under the action of gravity. However, there are two possible forces acting on the ball in the direction opposite to gravity, namely, buoyancy and air resistance. The contribution due to air resistance is proportional to the square of the velocity of the object. The equation of motion in this case is given by

### Table I. Description and physical characteristics of the balls used in the experiment.

<table>
<thead>
<tr>
<th>Ball</th>
<th>Description</th>
<th>Mass (gm)</th>
<th>Radius (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>smooth rubber ball</td>
<td>19.00</td>
<td>1.4</td>
</tr>
<tr>
<td>2</td>
<td>ping pong ball</td>
<td>1.75</td>
<td>1.9</td>
</tr>
<tr>
<td>3</td>
<td>nylon ball</td>
<td>5.50</td>
<td>1.9</td>
</tr>
</tbody>
</table>
where \( m_{\text{air}} \) is the mass of the air displaced by the ball and \( k \) is a proportionality constant. Because the dimensions of the balls are small, the volume and in turn the mass of air displaced are very small. Hence, contributions due to the buoyant force can be neglected, and the equation of motion reduces to

\[
md\frac{d^2s}{dt^2} = mg - kv^2.
\]

We note that the drag force on the falling body can be proportional to either its velocity given by the Navier-Stokes equation, \( F=6\pi\mu rv \), or the square of the velocity. Which dependence is more appropriate depends on whether the motion of the viscous medium around the falling body remains laminar or not, which is measured by the Reynolds number \( \text{Re} \),

\[
\text{Re} = \frac{d\rho v}{\mu}.
\]

where \( \mu \) is the viscosity of the medium of density \( \rho \) and \( d \) is the diameter of the ball. In our case the body is falling through (dry) air, whose viscosity is small (\( \approx 1.9 \times 10^{-5} \) kg/ms) with density 1.225 kg/m\(^3\). The Reynolds...
number is \( \approx 3000 \), that is, \( \text{Re} \gg 1 \), and hence the drag force on the body is

\[
F = \frac{1}{2} \rho A C_D v^2 \tag{6}
\]
or \( k = \frac{1}{2} C_D \rho A \), where \( A \) is the cross-sectional area and \( C_D \) is the drag coefficient. The drag coefficient depends on the shape, surface characteristics, and \( \text{Re} \). The value of \( C_D \) for smooth spheres as a function of \( \text{Re} \) is been given in Ref. 14.

The solution of Eq. (4) and its polynomial approximation is given in Ref. 4 and is

\[
s = \left( \frac{v_0^2}{g} \right) \ln \left[ \cosh \left( \frac{g t}{v_T} + \frac{v_0}{v_T} \sinh \left( \frac{g t}{v_T} \right) \right) \right], \tag{7a}
\]

\[
= v_0 t + \frac{g t^2}{2} - \frac{v_0^2}{v_T^2} - \frac{g^2 v_0^3}{3 v_T} \left( 1 - \frac{v_0^2}{v_T^2} \right), \tag{7b}
\]

where \( v_T = \sqrt{g m / k} \) is the terminal velocity and \( v_0 \) is the initial velocity of the falling body. The coefficient of \( t^2 \) is proportional to the rate of change of the acceleration.

For the conditions in which this experiment was done, the data points were fit to the third order polynomial in Eq. (7b). The results for the approximately 200 experiments lie on or between the two curves labeled I and II shown in Fig. 4. The fitted coefficients of \( t, t^2, \) and \( t^3 \) were used to determine the ball’s velocity \( v_1 \) as it crosses the first sensor where the ball has traveled at least 10 cm (10 cm from the drop tower’s top edge to the first sensor plus the unknown height from the edge from where the ball was dropped) and its acceleration \( g \). The balls are dropped from near the edge of the tower with zero initial velocity. Results for the plots shown in Fig. 4 are listed in Table II. The error in \( g \) is 1 part in \( 10^3 \) with as small as \( \pm 7 \mu s \) (max) error in the measurement of the time of flight.

The acceleration due to gravity in Delhi can be calculated using

\[
g = 9.780327 \left[ 1 + 0.005 302 4 \sin^2 L - 0.000 058 \sin^2 2L \right], \tag{8}
\]

where the latitude \( L \) of Delhi in radians is 0.499 455 (28° 37’ N). Equation (8) gives the acceleration due to gravity in Delhi as 9.7918 m/s². Our experimental value of \( g \) listed in Table II is in good agreement with the theoretical value. Notice that the velocity \( v_1 \) is different in each case (see Table II). This difference is expected because, even with the best of care, the balls were dropped from different heights.

The power series in Eq. (7b) suggests that a knowledge of the coefficients would enable us to estimate the ball’s terminal speed. However, if the ball’s initial velocity is zero, then the terms in the expansion associated with the terminal velocity also vanish. Fitting Eq. (7a) for the time dependence of the displacement is tedious due to the existence of logarithmic and hyperbolic terms. The relation between the velocity and the time for a body falling through a viscous medium is much simpler and is given by

\[
v = v_T \tanh \left[ \frac{g}{v_T} t + \tanh^{-1} \left( \frac{v_0}{v_T} \right) \right]. \tag{9}
\]

If \( v_0 = 0 \), Eq. (9) reduces to

\[
v = v_T \tanh \left[ \frac{g t}{v_T} \right]. \tag{10}
\]

Thus, to evaluate \( v_T \), we need to know how the velocity varies with time. To obtain this data, we need to go through a several more steps. For \( v_0 = 0 \) Eq. (7b) reduces to

\[
s = \frac{1}{2} g t^2 - b t^3. \tag{11}
\]

A fit to Eq. (11) yields the coefficients \( g \) and \( b \). The coefficients in

\[
v = g t - 3 b t^2, \tag{12}
\]

obtained by differentiating Eq. (11), would generate data for the time dependence of the velocity. A simpler way to obtain this dependence is to find the numerical derivative at regular intervals along the curves shown in Fig. 4. Both methods introduce a small error and the fitting as represented by the correlation factor (see Table II) was very good. The velocity variation with time obtained by either method can be fitted using Eq. (10). The terminal velocities obtained in this way are listed in Table III.

The data in Table III indicate that the terminal velocity, the time taken to attain this velocity, and the distance through which it has to travel to reach the terminal speed all depend on the object’s mass. The drag coefficient also depends on the surface area (and hence diameter). It would be useful to use balls with similar diameter and different mass. The empirical values of the drag coefficient \( k \) are also listed in Table III and are comparable to the value given in Ref. 14 (\( k = 3.8 \times 10^{-4} \text{Ns}^2/\text{m}^2 \)). Obtaining balls of different masses and similar dimensions was not easy. Our data analysis lets

Table II. Coefficients of the third order polynomial fits in Fig 4. The coefficents are the velocity of the ball as it passes the first LED-LDR pair, the acceleration due to gravity, and the rate of change of acceleration as the body falls through the viscous medium. The co-relation factors show how well the curves fit to the data.

<table>
<thead>
<tr>
<th>Ball</th>
<th>( v_1 ) (m/s)</th>
<th>( g ) (m/s²)</th>
<th>( b ) (m/s²)</th>
<th>co-relation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.559</td>
<td>9.7998</td>
<td>8.0e-9</td>
<td>0.998</td>
</tr>
<tr>
<td>II</td>
<td>1.559</td>
<td>9.7998</td>
<td>4e-9</td>
<td>0.998</td>
</tr>
<tr>
<td>2</td>
<td>1.363</td>
<td>9.789</td>
<td>0.643</td>
<td>0.999</td>
</tr>
<tr>
<td>II</td>
<td>1.368</td>
<td>9.7745</td>
<td>0.731</td>
<td>0.999</td>
</tr>
<tr>
<td>3</td>
<td>1.549</td>
<td>9.7998</td>
<td>0.418</td>
<td>0.998</td>
</tr>
<tr>
<td>II</td>
<td>1.445</td>
<td>9.7822</td>
<td>0.268</td>
<td>0.999</td>
</tr>
</tbody>
</table>

Table III. Estimated terminal velocity, time at which the ball attains terminal velocity, the coefficient \( k \), and the distance the ball travels before it attains terminal velocity.

<table>
<thead>
<tr>
<th>Ball</th>
<th>( v_T ) (m/s)</th>
<th>( t ) (s)</th>
<th>( s ) (m)</th>
<th>( k = mg/v_T^2 \times 10^{-4} \text{Ns}^2/\text{m}^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>35.11</td>
<td>3.58</td>
<td>62.9</td>
<td>1.27</td>
</tr>
<tr>
<td>II</td>
<td>35.11</td>
<td>3.58</td>
<td>62.9</td>
<td>1.27</td>
</tr>
<tr>
<td>2</td>
<td>6.32</td>
<td>0.58</td>
<td>1.77</td>
<td>4.3</td>
</tr>
<tr>
<td>II</td>
<td>5.89</td>
<td>0.53</td>
<td>1.52</td>
<td>4.92</td>
</tr>
<tr>
<td>3</td>
<td>7.97</td>
<td>0.74</td>
<td>2.87</td>
<td>8.47</td>
</tr>
<tr>
<td>II</td>
<td>9.97</td>
<td>0.94</td>
<td>4.6</td>
<td>5.4</td>
</tr>
</tbody>
</table>
us estimate the terminal velocity, the time taken to achieve it, and the distance through which the body has to travel before attaining it, even though the length of the drop tower is less than the distance through which the object needed to fall to attain $v_T$.

**ACKNOWLEDGMENTS**

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5 The circuit can be simplified by using a Schmitt trigger whose output is TTL compatible. This use would replace the op amp, zener diode, and its protective resistance.

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